The Mandelbrot Set

The Mandelbrot set is based on the recursive equation $z_{n+1} = z_n^2 + c$ where the pronumerals all represent complex numbers. The Mandelbrot set is the set of points c on the Argand plane such that given $z_0 = 0$, z_n remains bounded (does not go off to infinity) no matter how large n gets.

For example, if $c = 1 + 0i$ we get the following values for $z : z_0 = 0, z_1 = 0^2 + 1 = 1, z_2 = 1^2 + 1 = 2, z_3 = 2^2 + 1 = 5, z_4 = 5^2 + 1 = 26, \ldots$ which is unbounded, so $1 + 0i$ is outside the Mandelbrot set.

If $c = -1$, we get $z_0 = 0, z_1 = 0^2 - 1 = -1, z_2 = (-1)^2 - 1 = 0, z_3 = -1, z_4 = 0, \ldots$ which is bounded, so $-1 + 0i$ is in the Mandelbrot set.

If $c = i$, we get $z_0 = 0, z_1 = i, z_2 = -1 + i, z_3 = -i, z_4 = -1 + i, z_5 = -i \ldots$ which is bounded, etc.

There are many possible variations on this recursive equation, such as $z_{n+1} = z_n + c, z_{n+1} = cz_n, z_{n+1} = \frac{c}{z_n}, z_{n+1} = \sqrt{z_n} + c$.

For some of these it will be easy to predict which values of c will give bounded results. Others will be more challenging (or perhaps we should say 'interesting').

Investigate.